1 Derivation

Let us define $z_i(x)$ as the warp for epoch i at pixel x, and $z_c(x)$ as the coadd at that location. Then we have:

$$f_i = \sum_x z_i(x) \tag{1}$$

(summed over a top-hat aperture or weighted aperture) for the measured, uncorrected flux at each epoch, and

$$f_c = \sum_x z_c(x) \tag{2}$$

(again summed over an aperture) as an uncorrected measurement of the flux on the coadd.

The coadd pixels are related to the warp pixels by

$$z_c(x) = \sum_i w_i^c s_i z_i(x), \tag{3}$$

where w_i^c are the weights used in the coaddition process, and s_i are the per-epoch photometric scaling factors to put these on a common photometric system.

If we take the sum over x on both sides we get:

$$\begin{aligned} \sum_x z_c(x) &= \sum_x \sum_i w_i^c s_i z_i(x) && (4) \\ f_c &= \sum_i w_i^c s_i f_i. && (5) \end{aligned}$$

This is our uncorrected coadd flux, but we now want to correct this value to the standard passband. This correction requires knowledge of the spectral energy distribution (SED).

As shown in e.g. Burke, Rykoff et al. (2018), the AB flux in the observed passband for a single epoch i is given by:

$$f_i^{\mathrm{obs}} = \frac{\int_0^\infty F_\nu(\lambda) S_i^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda}{\int_0^\infty F^{\mathrm{AB}} S_i^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda}, \tag{6}$$

where $F_\nu(\lambda)$ is the SED, $S_i^{\mathrm{obs}}(\lambda)$ is the transmission function of the observed passband (which is a function of position, airmass, filter, etc.), and F^{AB} is a flat spectrum on the AB scale. In addition, as discussed in Burke, Rykoff et al. (2018), the integral in the denominator is the achromatic contribution to the throughput (essentially assuming a flat SED). Therefore, the scaling in the coadds is:

$$s_i = \frac{1}{c \int_0^\infty S_i^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda}, \tag{7}$$

where we have added an additional c term to denote the possibility of choosing an arbitrary coadd zeropoint. (Is this necessary to put in here?)

We similarly define the flux as would be observed through the (arbitrary) standard passband:

$$f^{\mathrm{std}} = \frac{\int_0^\infty F_\nu(\lambda) S^{\mathrm{std}}(\lambda)\lambda^{-1} d\lambda}{\int_0^\infty F^{\mathrm{AB}} S^{\mathrm{std}}(\lambda)\lambda^{-1} d\lambda}. \tag{8}$$

Note that the flux through the standard passband does not have a subscript i since it does not vary per epoch.

The warps have been created after scaling to the common reference zero-point, ignoring any chromatic terms. Therefore, in correcting the coadds, what we care about is the per-epoch "chromatic correction" which is given by the ratio of the flux in the observed passband to the standard passband, denoted r_i^{chrom}:

$$\begin{aligned} r_i^{\mathrm{chrom}} &\equiv f_i^{\mathrm{obs}} / f^{\mathrm{std}} &(9)\\ &= \frac{\int_0^\infty S^{\mathrm{std}}(\lambda)\lambda^{-1} d\lambda}{\int_0^\infty S_i^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda} \frac{\int_0^\infty F_\nu(\lambda) S_i^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda}{\int_0^\infty F_\nu(\lambda) S^{\mathrm{std}}(\lambda)\lambda^{-1} d\lambda}. &(10)\end{aligned}$$

However, this is rather inconvenient because we would prefer not to go back to the invidual epoch transmission curves and fluxes. Fortunately, we can compute the "coadded observed passband" $S_c^{\mathrm{obs}}(\lambda)$ (via analogy to the coadded point-spread-function (PSF)).

$$\begin{aligned} S_c^{\mathrm{obs}}(\lambda) &= \frac{\sum_i w_i^c s_i S_i^{\mathrm{obs}}(\lambda)}{\sum_i w_i^c} &(11)\\ S_c^{\mathrm{obs}}(\lambda) &= \frac{\sum_i w_i^c \frac{S_i^{\mathrm{obs}}(\lambda)}{\int_0^\infty S_i^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda}}{\sum_i w_i^c}, &(12)\end{aligned}$$

where we do the averaging at each wavelength step.

The corrected coadd flux then becomes:

$$\begin{aligned} f'_c &= \sum_i w_i^c f_i r_{\mathrm{obs}}^{\mathrm{chrom}} &(13)\\ &= f_c r_{\mathrm{obs}}^{\mathrm{chrom}} &(14)\\ &= f_c \frac{\int_0^\infty S^{\mathrm{std}}(\lambda)\lambda^{-1} d\lambda}{\int_0^\infty S_c^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda} \frac{\int_0^\infty F_\nu(\lambda) S_c^{\mathrm{obs}}(\lambda)\lambda^{-1} d\lambda}{\int_0^\infty F_\nu(\lambda) S^{\mathrm{std}}(\lambda)\lambda^{-1} d\lambda}. &(15)\end{aligned}$$

Therefore, given an SED we need to know the coadded transmission curve. In the "traditional" coadd, we need to know this per object because of the different inputs, so the amount of data to store is inconvenient and the advantage over recreating the full stack is limited. However, for cell-based coadds the entire cell will share the same observed transmission curve (assuming the sensor chromatic variation does not vary significantly on a ≈ 150 pixel cell size.[1]).

[1] That better be true.