Photometry using Compensated Filters

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What’s the Problem We’re Solving?

Let’s think about the model

$$I_i = S + F\phi(\boldsymbol{x}_i - \boldsymbol{x}_c) + \epsilon_i$$

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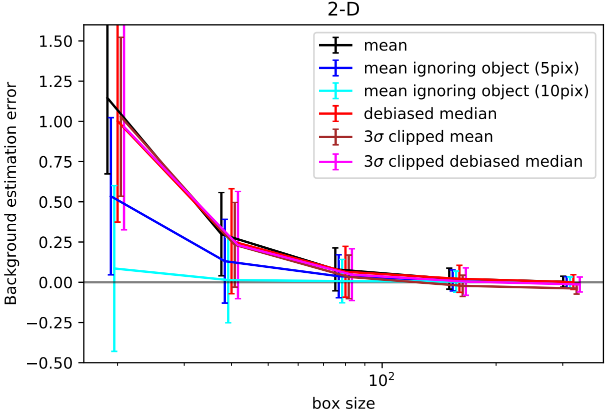

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where i is a pixel index, S is the background level (the "sky") and ϕ the PSF. One approach to estimating S is to calculate some statistic from the data. An alternative is to use a simultaneous MLE estimate for S and the flux F:

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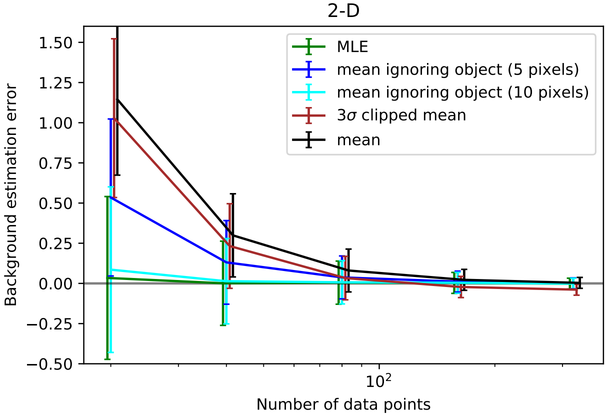

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There are other approaches (*e.g.* looking for the darkest parts of the image) and tweaks (*e.g.* subtracting the wings of bright stars), but this is how *e.g.* SDSS and Rubin estimate the sky.

Backgrounds in Stellar Surveys

Stellar surveys can use the same approach (*e.g.* SDSS), but it is also common to measure the background using an annulus near each object:

$$S = \frac{1}{\pi(R_2^2 - R_1^2)} \int_{R_1}^{R_2} I(\boldsymbol{x}) 2\pi r \, dr$$

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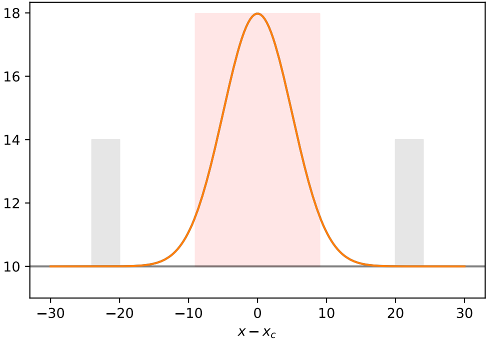

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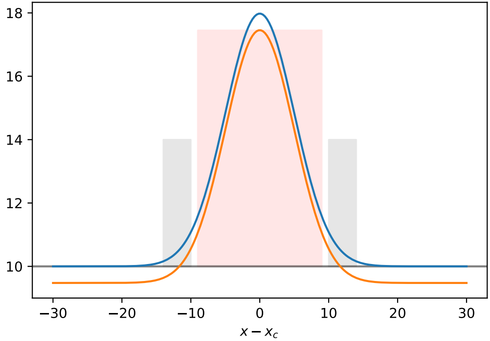

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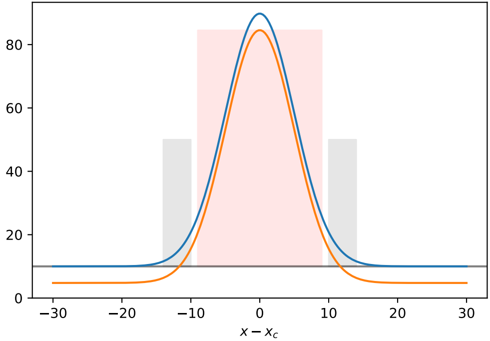

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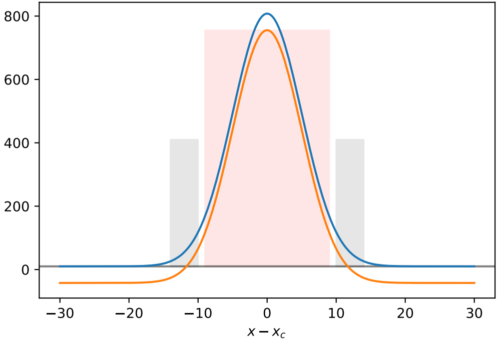

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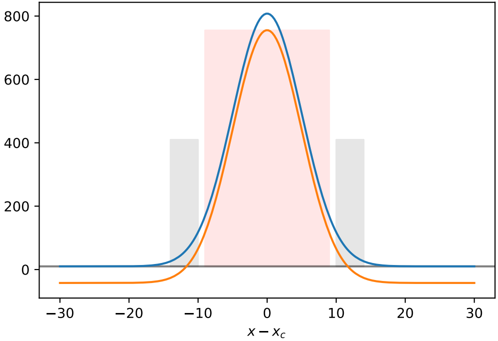

Note that the ratio of the true flux to the sky-subtracted flux is independent of the star's brightness.

Compensated Filters

We can rewrite this as a measurement filter:

$$\chi(\boldsymbol{x}; R_a p, R_1, R_2) = c \begin{cases} 1 & r \leq R_{ap} \\ -\frac{R_{ap}^2}{R_2^2 - R_1^2} & R_1 \leq r \leq R_2 \end{cases}$$

where c is chosen so that $\langle \int \chi \phi \, d^2\boldsymbol{x} \rangle = 1$, *i.e.* that $\int \chi I \, d^2\boldsymbol{x}$ is an unbiased estimator of F.

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As usual, the next question to address is about the estimator's variance.

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Let's write

$$\chi(\boldsymbol{x}; t, \beta) = c(t, \beta) \left(N(\boldsymbol{x}_c, \beta^2) - N(\boldsymbol{x}_c, t^2\beta^2) \right)$$

Variances for Gaussian Compensated Filters

If the PSF is $N(0, \alpha^2)$ and we take the background noise σ^2 to dominate the object's Poisson noise,

$$c(t, \alpha) = 4\pi\alpha^2 \left(\frac{t^2 + 1}{t^2 - 1}\right)$$

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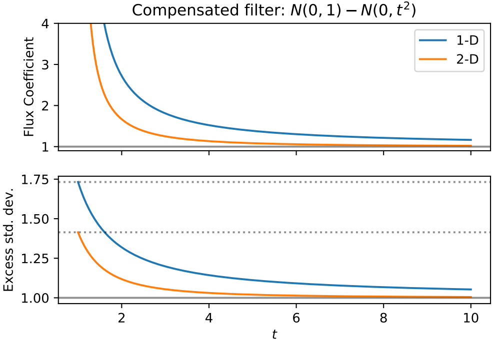

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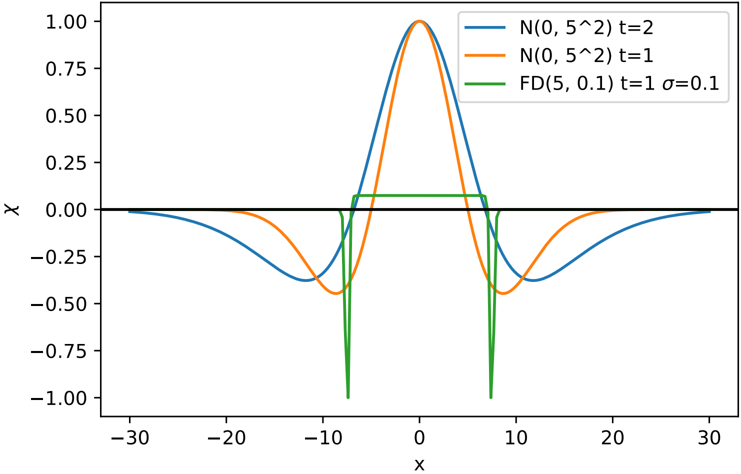

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I'm not sure if this is what we want to do.

The End